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NON-ABELIAN SUPERCONDUCTORS - LESSONS FROM SUPERSYMMETRIC GAUGE THEORIES FOR QCD

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Abstract:

Much about the confinement and dynamical symmetry breaking in QCD might be learned from models with supersymmetry. In particular, models based on $N = 2$ supersymmetric theories with gauge groups $SU(N)$, $SO(N)$ and $USp(2N)$ and with various number of flavors, give deep dynamical hints about these phenomena. For instance, the BPS non-abelian monopoles can become the dominant degrees of freedom in the infrared due to quantum effects. Upon condensation (which can be triggered in these class of models by perturbing them with an adjoint scalar mass) they induce confinement with calculable pattern of dynamical symmetry breaking. This may occur either in a weakly interacting regime or in a strongly coupled regime (in the latter, often the low-energy degrees of freedom contain relatively non-local monopoles and dyons simultaneously and the system is near a nontrivial fixed-point). Also, the existence of sytems with BPS *non-abelian vortices* has been shown recently. These results point toward the idea that the ground state of QCD is a sort of dual superconductor of non-abelian variety.

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1 Confinement in $SU(N)$ YM Theory

The test charges in $SU(N)$ YM theory take values in $(Z_N^{(M)}, Z_N^{(E)})$ where Z_N is the center of $SU(N)$ and $Z_N^{(M)}, Z_N^{(E)}$ refer to the magnetic and electric center charges. $(Z_N^{(M)}, Z_N^{(E)})$ classification of phases follows [1, 2]. (See Figures) Namely,

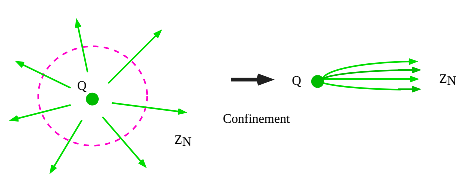
1. If field with $x = (a, b)$ condense, particles $X = (A, B)$ with
$$\langle x, X \rangle \equiv a\, B - b\, A \neq 0 \quad (mod\ N)$$
are confined. (e.g. $\langle \phi_{(0,1)} \rangle \neq 0 \rightarrow$ Higgs phase.)

2. Quarks are confined if some field χ exist, such that
$$\langle \chi_{(1,0)} \rangle \neq 0.$$

3. In the softly broken $N = 4$ (to $N = 1$) theory (often referred to as $N = 1^*$) all different types of massive vacua, related by $SL(2, Z)$, appear; the chiral condensates in each vacua are known.

4. *Confinement index* [3] is equal to the smallest possible $r \in Z_N^{(E)}$ for which Wilson loop displays no area law. For instance, for $SU(N)$ YM, $r = N$ in the vacuum with complete confinement; $r = 1$ in the totally Higgs vacuum, etc.

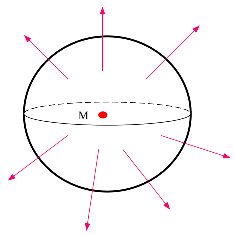
5. In softly broken $N = 2$ Gauge Theories, dynamics turns out to be particularly transparent.



The questions we wish to address here are: What is χ in QCD? How do they interact ? Is chiral symmetry breaking related to confinement? θ vacua?; $\frac{\epsilon'}{\epsilon}$; $\Delta I = \frac{1}{2}$?

A familiar idea is that the ground state of QCD is a dual superconductor [2]. There exist no elementary nor soliton monopoles in QCD; however, monopoles can be detected as topological singularities (lines in $4D$) of Abelian gauge fixing, $SU(3) \rightarrow U(1)^2$. Alternatively, one can assume that certain configurations close to the Wu-Yang monopole ($SU(2)$)

$$A_\mu^a = \tilde{\sigma}(x)(\partial_\mu n \times n)^a + \ldots, \qquad n(r) = \frac{r}{r} \quad \Rightarrow \quad A_i^a = \epsilon_{aij}\frac{r^j}{r^3}$$

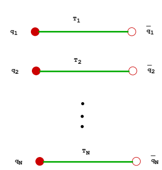


$\Pi_1(SU(N)/Z_N) \sim Z_N$

dominate [4]. Although there are some evidence in lattice QCD [5] for "Abelian dominance", there are several questions to be answered. Do abelian monopoles carry flavor? What is the form of L_{eff}? What about the gauge dependence? Most significantly, does dynamical $SU(N) \to U(1)^{N-1}$ breaking occur? That would imply [6] a richer spectrum of mesons ($T_1 \neq T_2$, etc) not seen in Nature and not expected in QCD. Both in Nature and presumably in QCD there is only one "meson" state

$$\text{Meson} \sim \sum_{i=1}^{N} |\, q_i\, \bar{q}_i \,\rangle$$

i.e., 1 state vs $\left[\frac{N}{2}\right]$ states. (See Figure.) Assuming $SU(N) \to U(1)^{N-1} \times Weyl$ symmetry is not enough to solve the problem.



In an attempt to answer these questions we address ourselves to the world of supersymmetric gauge theories, where the non-abelian gauge dynamics can be analyzed to depth. Supersymmetric theories in fact continue to surprise us for rich insight they give us about the dynamics of non-abelian interactions. Although not a main subject of this talk, let me point out that much is going on at present in the exploration of various $N = 1$ or $N = 2$ susy gauge theories [7].

Let us start by recalling some basic facts about supersymmetric gauge theories [8].

2 SQCD

2.1 Basics of Susy gauge theories

The basic susy algebra contains

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha,\dot\alpha} P_\mu.$$

In order to construct supersymmetric theories it is convenient to introduce superfields [8]

$$F(x,\theta,\bar\theta) = f(x) + \theta\psi(x) + \ldots$$

$$Q_\alpha = \partial ø \partial\theta^\alpha - i\sigma^\mu_{\alpha,\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu, \quad \bar{Q}_{\dot\alpha} = \partial ø \partial\bar\theta^{\dot\alpha} - i\theta^\alpha\sigma^\mu_{\alpha,\dot\alpha}\partial_\mu,$$

In general they are reducible with respect to supersymmetry transformations. We construct smaller irreducible multiplets. Chiral superfields are defined by the constraint $\bar{D}\Phi = 0$ ($D\Phi^\dagger = 0$) so that

$$\Phi(x,\theta,\bar\theta) = \phi(y) + \sqrt{2}\theta\,\psi(y) + \theta\theta\,F(y), \quad y = x + i\theta\sigma\bar\theta$$

$$D_\alpha = \partial ø \partial\theta^\alpha + i\sigma^\mu_{\alpha,\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu, \qquad \bar{D}_{\dot\alpha} = -\partial ø \partial\bar\theta^{\dot\alpha} - i\theta^\alpha\sigma^\mu_{\alpha,\dot\alpha}\partial_\mu,$$

Verctor superfields are defined to be real $V^\dagger = V$. They are conveniently expressed in terms of a chiral (fermionic) superfield

$$W_\alpha = -1ø4\bar{D}^2 e^{-V} D_\alpha e^V = -i\lambda \,+\, \mu, ø2\,(\sigma^\mu\,\bar\sigma^\nu)^\beta_\alpha\, F_{\mu\nu}\,\theta_\beta + \,\ldots$$

Supersymmetric Lagrangian ($\int d\theta_1\,\theta_1 = 1$, etc) can then be written simply as

$$\mathcal{L} = \frac{1}{8\pi}\mathrm{Im}\,\tau_{cl}\left[\int d^4\theta\,\Phi^\dagger e^V\,\Phi + \int d^2\theta\,1ø2WW\right] + \int d^2\theta\,W(\Phi) \qquad (2.1)$$

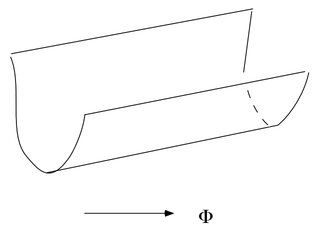
where $W(\Phi)$ is the superpotential and $\tau_{cl} = \theta ø 2\pi + 4\,\pi\,i ø g^2$. The scalar potential is the sum of the F-term and D-term:

$$V_{sc} = \sum_{mat} |\partial W ø \partial\phi|^2 + 1ø2\sum_a \left|\sum_{mat}\phi^* t^a \phi\right|^2.$$

For SQCD, $\{\Phi\} \to Q \sim \underline{N}$, $\tilde{Q} \sim \underline{N}^*$ of $SU(N)$

$$G_F = SU(n_f) \times SU(n_f) \times U_V(1) \times U_A(1) \times U_\lambda(1).$$

The theory has a characteristic continuous vacuum degeneracy - flat directions or classical moduli space (CMS). For instance, it looks like for $n_f < n_c$,



$$Q = \tilde{Q}^\dagger = \begin{pmatrix} a_1 & 0 & \dots & 0 \\ 0 & \ddots & & \\ 0 & \dots & & a_{n_f} \\ 0 & 0 & \dots & 0 \\ \dots & & & \dots \end{pmatrix}.$$

The problem is: is superpotential generated dynamically? Is CMS modified?

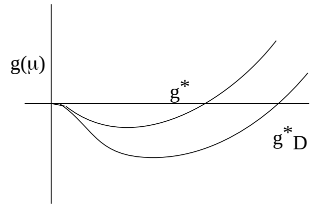
2.2 Phases of SQCD; Seiberg's duality

The analyses by use of various supersymmetry, Ward-Takahashi identities, nonrenormalization theorems [9], symmetry considerations [10], dynamical (instanton) calculations [11, 12] and Seiberg's duality [13] have established the following picture of the vacua in the massless SQCD:

- The dynamically generated superpotential implies the vacuum runaway for $n_f < n_c$, whereas no superpotential is generated for $n_f > n_c$.
- For $n_f = n_c$ the moduli space (space of vacua) is quantum mechanically modified as

$$\det M - B\,\tilde{B} = \Lambda^{2n_f}.$$

- For $3n_c ø 2 < n_f < 3n_c$ (conformal window), the system at the origin of the moduli space is in an infrared fixed point (superconformal theory - SCFT): the low energy phsyics is described either as the original SQCD or as the dual $SU(\tilde{n}_c) = SU(n_f - n_c)$ theory with dual quarks.



N_f	Deg.Freed.	Eff. Gauge Group	Phase	Symmetry
0 (SYM)	-	-	Confinement	-
$1 \leq N_f < N_c$	-	-	no vacua	-
N_c	$M, B, \tilde{B}$	-	Confinement	$U(N_f)$
$N_c + 1$	$M, B, \tilde{B}$	-	Confinement	Unbroken
$N_c + 1 < N_f < \frac{3N_c}{2}$	$q, \tilde{q}, M$	$SU(\tilde{N}_c)$	Free-magnetic	Unbroken
$\frac{3N_c}{2} < N_f < 3N_c$	$q, \tilde{q}, M$ or $Q, \tilde{Q}$	$SU(\tilde{N}_c)$ or $SU(N_c)$	SCFT	Unbroken
$N_f = 3N_c$	$Q, \tilde{Q}$	$SU(N_c)$	SCFT (finite)	Unbroken
$N_f > 3N_c$	$Q, \tilde{Q}$	$SU(N_c)$	Free Electric	Unbroken

3 Confining vacua in $N = 2$ Supersymmetric Gauge Theories

Although dynamical properties of $N = 1$ supersymmetric gauge thoeries are thus quite well known by now, as in the example of SQCD discussed above, a detailed understanding of the working of confinement and dynamical symmmetry breaking is still lacking. $N = 2$ theories appear to allow for a deeper level of understanding of the nonperturbative dynamics, by displaying the quantum behavior of magnetic monopoles and vortices very clearly. We start from afar: Dirac's monopoles in QED.

3.1 Dirac's monopoles

As is well known QED admits pointlike magnetic monopoles if Dirac's quantization condition

$$g\, e = \frac{n}{2}, \qquad n \in \ Z, \tag{3.1}$$

is satisfied. In the presence of a magnetic monopole, there cannot be a gauge vector potential which is everywhere regular. A possible singularity (Dirac string) along $(0, 0, 0) \to (0, 0, -\infty)$ is invisible if (3.1) is satisfied. A proper formulation is to cover S^2 by two regions a $(0 \leq \theta < \frac{\pi}{2} + \epsilon)$ and b $(\frac{\pi}{2} - \epsilon < \theta \leq \pi)$ [14]

$$(A_\phi)^a = \frac{g}{r \sin\theta}(1 - \cos\theta), \qquad (A_\phi)^b = -\frac{g}{r \sin\theta}(1 + \cos\theta),$$

so that in each neighborhood the vector potential is regular. The two descriptions are related along the equator by a gauge transformation

$$A_i^a = A_i^b - U^\dagger \frac{i}{e} \partial_i U, \qquad U = e^{2ige\phi}.$$

The gauge transformation is well-defined if the condition (3.1) is met. More generally, for dyons (e_1, g_1), $(e_2 g_2)$, the quantization condition reads

$$e_1\, g_2 - e_2\, g_1 = \frac{n}{2}, \qquad n \in \ Z. \tag{3.2}$$

The topology involved is: $\Pi_1(U(1)) = \ Z$.

3.2 Non-Abelian gauge theories

In the case of a non-abelian gauge group, one might embed Dirac's monopole in a $U(1)$ subgroup. However, the homotopy group properties such as

$$SU(2) \sim S^3, \qquad \Pi_1(SU(2)) = \ 1, \tag{3.3}$$

$$SO(3) \sim \frac{S^2}{Z_2}, \qquad \Pi_1(SO(3)) = \ Z_2, \tag{3.4}$$

show that there are no topologically stable monopoles in $SU(2)$, $SU(N)$; there is only one type of monopole in $SO(3)$, and so on [14].

In spontaneously broken gauge theories, instead, there are ('t Hooft-Polyakov) monopoles [15]

$$SU(2) \stackrel{\langle\phi\rangle\neq 0}{\longrightarrow} U(1)$$

$$D\phi \stackrel{r\to\infty}{\longrightarrow} 0, \quad \Rightarrow \quad \phi \sim U \cdot \langle\phi\rangle \cdot U^{-1};$$

$$A_i \sim U \cdot \partial_i U^\dagger \quad \Rightarrow \quad F_{ij} = \epsilon_{ijk}\frac{r_k}{r^3} m \frac{\tau_3}{2}$$

($m = \pm 1, \pm 2, \ldots$) which are regular, finite energy soliton-like configurations, with topology, $\Pi_2(SU(2)/U(1)) = \Pi_1(U(1)) = \ Z$. The static energy can be written as (Bogomolny)

$$H = \int d^3x \left[\frac{1}{4}(F^a_{ij})^2 + \frac{1}{2}(D_i\phi^a)^2 + \frac{\lambda}{2}(\phi^2 - v^2)^2\right].$$

$$= \int d^3x \left[\frac{1}{4}(F^a_{ij} - \epsilon_{ijk}D_k\phi^a)^2 + \frac{1}{2}F^a_{ij}\epsilon_{ijk}D_k\phi^a + \text{pot.}\right]$$

where $\frac{1}{2}F^a_{ij}\epsilon_{ijk}D_k\phi^a = \partial_i S_i;\ \ S_i = \frac{1}{2}\epsilon_{ijk}F^a_{ij}\phi^a$: the second term in the square bracket is a topological invariant. It follows that in a given sector

$$H \geq \int d^3x \nabla \cdot S = \frac{4\pi v}{g} m, \qquad m = 1, 2, \ldots$$

If $\lambda = 0$ the configuration of the minimum energy is given by the solution of the linear (Bogomolny) equations

$$F^a_{ij} - \epsilon_{ijk}D_k\phi^a = 0; \qquad B^a_i = D_i\phi^a$$

whose solutions are known in analytic form.

A more general situation is that of a spontaneously broken gauge theory

$$G \stackrel{\langle\phi\rangle\neq 0}{\longrightarrow} H$$

where H is non-abelian [16, 17]. The asymptotic behavior is

$$D\phi \stackrel{r\to\infty}{\longrightarrow} 0, \quad \Rightarrow \quad \phi \sim U \cdot \langle\phi\rangle \cdot U^{-1} \sim \Pi_2(G/H) = \Pi_1(H);$$

$$A_i \sim U \cdot \partial_i U^\dagger \rightarrow \quad F_{ij} = \epsilon_{ijk} \frac{r_k}{r^3} \beta_\ell T_\ell .$$

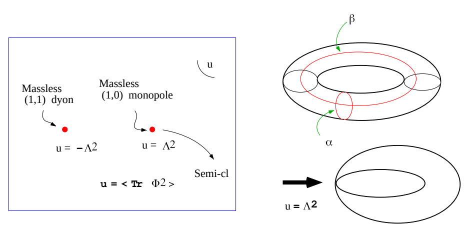
where $T_i \in$ Cartan S.A. of H. Topological quantization leads to $2\ \alpha \cdot \ \beta \subset \ Z$, where $\beta_i =$ weight vectors of $\tilde{H} =$ dual of H. Examples of the dual of several groups are given in the Table below.

Table 1: $\tilde{H} \Leftrightarrow H$

$SU(N)/Z_N$	$\Leftrightarrow$	$SU(N)$
$SO(2N)$	$\Leftrightarrow$	$SO(2N)$
$SO(2N+1)$	$\Leftrightarrow$	$USp(2N)$

The point of this discussion is that these objects – abelian and non-abelian BPS monopoels – appear naturally in $N=2$ gauge theories as dynamical degrees of freedom and play a crucial role in the infrared physics. Before coming to this central issue, however, let us first briefly review the celebrated Seiberg-Witten solution of $N=2$ gauge theories [18, 19, 20].

3.3 Seiberg-Witten, $N=2$ Gauge Theories



The Lagrangian of a $N=2$ YM theory is Eq.(2.1) with $W(\Phi)=0$, where Φ is a chiral superfield in the adjoint representation of the gauge group. For $SU(2)$ the vacuum degeneracy (moduli space) is parametrized as

$$\langle \Phi \rangle = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \tag{3.5}$$

$a \neq 0$ breaks the gauge symmetry as $SU(2) \rightarrow U(1)$. In the infrared,

$$L_{eff} = \mathrm{Im} \left[\int d^4\theta \, \bar{A} \, \frac{\partial F_p(A)}{\partial A} + \int \frac{1}{2} \frac{\partial^2 F_p(A)}{\partial A^2} W_\alpha W^\alpha \right]$$

where W_α, A describe $N=2$, $U(1)$ theory. $F_p(A)$ is called prepotential. Define the dual of A, $A_D \equiv \frac{\partial F_p(A)}{\partial A}$: then

$$\frac{d\,A_D}{d\,u} = \oint_\alpha \frac{dx}{y}, \qquad \frac{d\,A}{d\,u} = \oint_\beta \frac{dx}{y},$$

where the curve ($u \equiv \mathrm{Tr}\,\langle \Phi^2 \rangle$ describes the quantum moduli space - QMS) is

$$y^2 = (x-u)(x+\Lambda^2)(x-\Lambda^2).$$

The exact mass formula (BPS) following from the $N=2$ susy algebra is

$$m_{n_m,n_e} = \sqrt{2}\,|\, n_m A_D + n_e A \,|.$$

The above four formulae consitute the Seiberg-Witten solution for the pure $N=2$ Yang-Mills theory [18]. The solution has been extended to more general gauge theories [19, 20].

The adjoint scalar mass ($\mu\,\Phi^2$ perturbation) leads to the low-energy effective superpotential near the singularity, $u \simeq \Lambda^2$:

$$W_{eff} = \sqrt{2}\,A_D\,M\tilde{M} + \mu\,U(A_D)$$

Minimization of the potential leads to the condensation of the monopole $\langle M \rangle \sim \sqrt{\mu\,\Lambda}$ (Confinement).

It is interesting to note that at the singularities $u = \pm\Lambda^2$, instanton sum diverges

$$\langle \mathrm{Tr}\Phi^2 \rangle = \frac{a^2}{2} + \frac{\Lambda^4}{a^2} + \ldots = \ldots + 1 + 1 + 1 + \ldots$$

The discussion can be generalized to $N=2$ pure YM theories with other gauge groups. In general, dynamical abelianization occurs near the monopole singularities, for instance, $SU(N)$ gauge group gets dynamically broken as $SU(N) \to U(1)^{N-1}$ (*cfr* QCD).

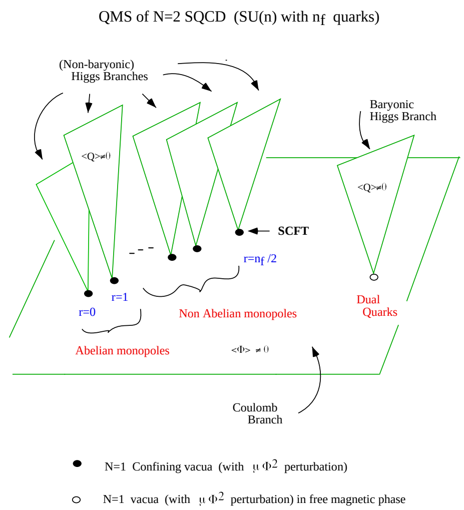
It is important to realize that these light "monopoles" are indeed 't Hooft-Polyakov monopoles becoming light by quantum corrections. This can be proven by studying the charge fractionalization [21, 22]. For instance, the electric charge of the monopole is known to behave as

$$\frac{2}{g} Q_e = n_e + \left[-\frac{4}{\pi} \mathrm{Arg}\, a + \frac{1}{2\pi} \sum_{f=1}^{N_f} \mathrm{Arg}\,(m_f^2 - 2a^2) \right] n_m + \ldots$$

in the semi-classical region where a is the adjoint scalar VEV. The Seiberg-Witten exact solution, when extrapolated back to the semi-classical domain, reproduces exactly this results. An analogous check has been done for the quark-number fractionalization [21]. There is an interesting phenomenon of quantum quenching of quark numbers of massless, condensing monopole. Also, the non-abelian flavor quantum numbers of the monopoles as encoded in the Seiberg-Witten solution are consistent with the well-known Jackiw-Rebbi mechanism.

3.4 More general $N = 2$ models

The study of the more general class of $N = 2$ theories [20, 23, 24] has shown that there are variety of confining vacua (see Fig. 3.4):

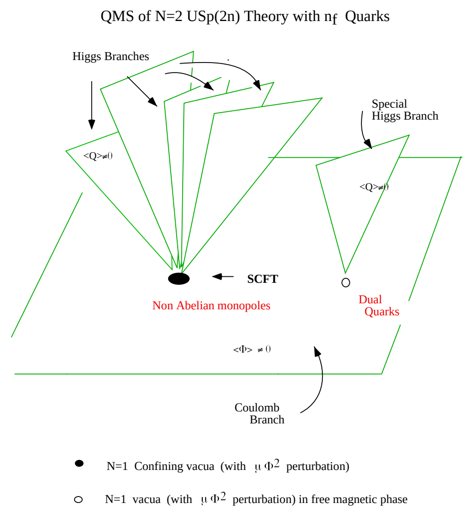


1. There are vacua ($r = 0, 1$) in which the low-energy effective action is an abelian (dual) gauge theory. Upon the adjoint scalar mass perturbation $\mu\,\Phi^2$, the magnetic monopole condenses (confinement); the system displays dynamical abelianization, a feature not shared by the real world QCD;

2. In a series of r-vacua, the effective action is $G_{eff} \sim SU(r) \times U(1)^{n_c - r - 1}$; with n_f dual quarks in $\underline{r}$ of the low-energy $SU(r)$ group. The "dual quarks" can be identified with the standard non-abelian monopoles [25]. These r-vacua exist for $r \leq \frac{n_f}{2}$;

3. Superconformal theory (SCFT) occurs at $r = \frac{n_f}{2}$. Here the question is what (mutually nonlocal) degrees of freedom describe the SCFT (which confines upon the perturbation $\mu\,\Phi^2$).

Physics of $USp(2\,n_c)$ ($SO(n_c)$ similar) theory is also very interesting. All r vacua at finite m are identical to those in the $SU(n_c)$ theory. They collapse into a single SCFT at $m \to 0$; in other words, all confining vacua of the theory with a vanishing bare quark masses (with perturbation $\mu\,\Phi^2$) are deformed SCFT, with mutually nonlocal dyons in the infrared [24]. Also, the global symmetry breaking pattern ($SO(2\,n_f) \to U(n_f)$ in the case of $USp(2\,n_c)$ theory) in these deformed SCFT is very reminiscent of what happens in QCD (cfr $\langle \bar{\psi}\,\psi \rangle^{(QCD)} \neq 0$).



3.5 More about non-abelian Monopoles

Consider the system with gauge symmetry breaking

$$SU(3) \xrightarrow{\langle\phi\rangle} SU(2) \times U(1), \qquad \langle\phi\rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & -2v \end{pmatrix}$$

By making use of the 't Hooft-Polyakov solutions in $SU_U(2), SU_V(2) \subset SU(3)$ one finds two degenerate $SU(3)$ solutions. Analogously, for the system with symmetry breaking

monopoles	$\tilde{SU}(2)$	$\tilde{U}(1)$
$\tilde{q}$	$\underline{2}$	1

$$SU(n) \xrightarrow{\langle\phi\rangle} SU(r) \times U^{n-r}(1), \qquad \langle\phi\rangle = \begin{pmatrix} v_1 \mathbf{1}_{r\times r} & 0 & \dots & 0 \\ 0 & v_2 & 0 & \dots \\ 0 & 0 & \ddots & \dots \\ 0 & 0 & \dots & v_{n-r+1} \end{pmatrix}$$

one finds the following set of the minimal monopoles:

We note [25] that they represent a degenerate r-plet of monopole solutions (q); that they have *the same charge structure as that of the "dual quarks" appearing in the* r*-vacua of* $N=2$ *SQCD*. The latter have also the correct flavor quantum numbers as expected from the Jackiw-Rebbi mechanism.

monopoles	$\tilde{SU}(r)$	$\tilde{U}_0(1)$	$\tilde{U}_1(1)$	$\tilde{U}_2(1)$	$\ldots$	$\tilde{U}_{n-r-1}(1)$
q	$\underline{r}$	1	0	0	$\ldots$	0
e_1	$\underline{1}$	0	1	0	$\ldots$	0
e_2	$\underline{1}$	0	0	1	0	0
$\vdots$	$\underline{1}$	0	$\ldots$			0
e_{n-r-1}	$\underline{1}$	0	0	$\ldots$	$\ldots$	1

3.6 Subtle are non-abelian monopoles

There are certain subtleties about the non-abelian monopoles. First of all, "colored dyons" are known not to exist [26]. More precisely, in the background of a 't Hooft-Polyaknow monpole it is not possible to define a globally defined set of generators isomorphic to those of H. This being so, it does not preclude the non-abelian monopoles of our interest: magnetic particles having abelian and non-abelian charges, *both* magnetic, can perfectly well exist, and do appear in the $r-$ vacua of the sofly broken $N = 2$ SQCD [25]. The main point is that the non-abelian monopoles are multiplets of the dual $\tilde{H}$ group, not of H itself. Transformations among the members of a multiplet of monopoles are nonlocal transformations with respect to the original gauge field variables.

The no-go theorem nonetheless means that the true gauge symmetry of the theory is not $H \otimes \tilde{H}$, as sometimes suggested, but $\tilde{H}$, H or some other combination, according to which physical degrees of freedom are relevant in a particular situation.

Also, it is not justified to study the system $G \stackrel{\langle\phi\rangle\neq 0}{\longrightarrow} H$ as a limit of maximally broken cases as sometimes made in the literature. non-abelian monopoles are never really semi-classical, even if

$$\langle\phi\rangle \gg \Lambda_H.$$

For H were broken it would produce simply an *approximately* degenerate set of monopoles as, for instance, in the pure $N = 2$, $SU(3)$ theory. Only if H remains unbroken do non-abelian monopoles in irreducible representation of $\tilde{H}$ appear. Which option is realized is a dynamical question which cannot be determined from a semiclassical consideration.

The important fact is that the second option *is* realized in the r vacua of $N = 2$ SQCD with $SU(r) \times U(1)^{n_c - r+1}$ gauge group, where $r < \frac{N_f}{2}$. This last constraint can be understood from a renormalization-group consideration: for $r < \frac{N_f}{2}$ there is a sign flip in the beta functions of the dual magnetic gauge group, with respect to that in the underlying theory:

$$b_0^{(dual)} \propto -2\,r + n_f > 0, \qquad b_0 \propto -2\,n_c + n_f < 0.$$

In fact, when such a sign flip is not possible, *e.g.*, pure $N = 2$ YM, dynamical abelianization occurs! *The quantum behavior of non-abelian monopoles thus depends critically on the presence of massless fermions in the underlying theory.* $r = \frac{n_f}{2}$ is a boundary case: the corresponding vacua are SCFT (nontrivial IR fixed point). Non-abelian monopoles and dyons still show up as low-energy degrees of freedom, but their interactions are nonlocal and strong. The possible mechanism of confinement in these vacua has been recently studied [27].

3.7 $\mathbf{Z}_N$ Vortices

Once the relevant degrees of freedom which act as the order parameter of confinement are identified, we are interested in the dominant field configurations which are capable of actually confining quarks. In the abelian dual superconductor picture of confinement in a $SU(N)$ YM theory, the quarks would be confined by abelian Abrikosov-Nielsen-Olesen vortices of $U(1)^{N-1}$. However, this leads to the difficulty mentioned at the end of the Section 1. The quarks must be confined by some sort of non-abelian chromoelectric vortices.

The simplest type of vortices involving a non-abelian gauge group is the $\mathbf{Z}_N$ vortex, which occurs in a system with gauge symmetry breaking

$$SU(N) \Rightarrow \mathbf{Z}_N.$$

An analogous vortex appear in a system with a general symmetry breaking pattern, $H \Rightarrow C$, a discrete center. Vortices represent nontrivial elements of $\Pi_1(H/C)$, e.g. $\Pi_1(SU(N)/\mathbf{Z}_N) = \mathbf{Z}_N$. The asymptotic behavior of the fields is

$$A_i \sim \frac{i}{g} U(\phi) \partial_i U^\dagger(\phi); \quad \phi_A \sim U \phi_A^{(0)} U^\dagger, \qquad U(\phi) = \exp i \sum_j^r \beta_j T_j \phi$$

where T_j are the generators of the Cartan subalgebra of H. The quantization condition is ($\alpha =$ root vectors of H)

$$U(2\pi) \in \mathbf{Z}_N, \qquad \alpha \cdot \beta \in \mathbf{Z} :$$

the vortices are characterized by the *weight vectors* of the group $\tilde{H}$, dual of H. It seems as though the vortex solutions were classified according to the irreducible representations of $\tilde{H} = SU(N)$. Actually, the fact that the topology involved is $\Pi_1(SU(N)/\mathbf{Z}_N) = \mathbf{Z}_N$ means that the stable vortices are characterized by $\mathbf{Z}_N$ charge (N-ality) only [28].

These $\mathbf{Z}_N$ vortices are non BPS and this makes the analysis of these objects so far relatively little explored. However there are interesting quantities which characterize these systems such as the tension ratios for different N-ality sources: an intriguing proposal (sine

formula) [29, 30] is

$$T_k \propto \sin \frac{\pi\, k}{N}$$

which can be measured on the lattice.

3.8 BPS vortices; non-abelian Superconductors

Systems with BPS vortices with a non-abelian flux - non-abelian superconductors - have been shown to exist only recently [32][1]. Consider a gauge theory in which the gauge group is broken at two very different scales

$$G \stackrel{\langle\phi\rangle\neq 0}{\longrightarrow} H \stackrel{\langle\phi'\rangle\neq 0}{\longrightarrow} \emptyset, \qquad \langle\phi\rangle \gg \langle\phi'\rangle,$$

where the unbroken (non-Abelian) group H gets broken completely at a much lower scale, $\langle\phi'\rangle$. We are interested in the physics at scales between the two scales $\langle\phi\rangle$ and $\langle\phi'\rangle$. When $\Pi_1(H) \neq \emptyset$ the system develops vortices. If the theory contains an exact continuous symmetry G_F, respected both by the interactions and by the vacuum (not spontaneously broken), but broken by a vortex solution, then there will be a nontrivial degeneracy of vortex solutions (zero modes).

An example [32] is the $SU(3)$ $N=2$ theory with $n_f = 4, 5$ quark flavors with large common (bare) mass m, with the $N=2$ symmetry broken softly to $N=1$ by the adjoint mass term, $\mu \,\mathrm{Tr}\, \Phi^2$. We consider a particular vacuum, the "$r=2$" vacuum, of this system, which is characterized by the VEVs

$$\Phi = -\frac{1}{\sqrt{2}} \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & -2m \end{pmatrix}, \qquad < q^{kA} >=< \bar{\tilde{q}}^{kA} >= \sqrt{\frac{\xi}{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

where $\xi = \mu\, m$. We take the (bare) quark mass m much larger than μ so that $m \gg \sqrt{\xi}$. At the mass scales between m and $\sqrt{\xi}$, the system has an exact $SU(2) \times U(1)/Z_2$ gauge symmetry as well as an $SU(n_f)$ global symmetry. The action has the form, after the Ansatz $\Phi = \langle\Phi\rangle$; $q = \tilde{q}^\dagger$; and $q \to \frac{1}{2} q$:

$$S = \int d^4x \left[\frac{1}{4g_2^2} \left(F_{\mu\nu}^a\right)^2 + \frac{1}{4g_1^2} \left(F_{\mu\nu}^8\right)^2 + \left|\nabla_\mu q^A\right|^2 \right.$$
$$\left. + \frac{g_2^2}{8} \left(\bar{q}_A \tau^a q^A\right)^2 + \frac{g_1^2}{24} \left(\bar{q}_A q^A - 2\xi\right)^2 \right]. \tag{3.6}$$

[1] An apparently similar, but different model has been studied by Hanany and Tong [31].

The tension can be rewritten à la Bogomolny:

$$
\begin{aligned}
T &= \int d^2x \Big(\sum_{a=1}^{3} \left[\frac{1}{2g_2} F^{(a)}_{ij} \pm \frac{g_2}{4} \Big(\bar{q}_A \tau^a q^A \big) \epsilon_{ij} \right]^2 + \\
&+ \left[\frac{1}{2g_1} F^{(8)}_{ij} \pm \frac{g_1}{4\sqrt{3}} \left(|q^A|^2 - 2\xi \right) \epsilon_{ij} \right]^2 + \frac{1}{2} \left| \nabla_i \, q^A + i \, \varepsilon \, \epsilon_{ij} \nabla_j \, q^A \right|^2 \pm \frac{\xi}{2\sqrt{3}} \tilde{F}^{(8)} \Big)
\end{aligned}
$$

where the first three terms are positive definite and the fourth term is a topologically invariant, $U(1)$ flux. The non-abelian Bogomolny equations

$$
\begin{aligned}
&\frac{1}{2g_2} F^{(a)}_{ij} \pm \frac{g_2}{4} \Big(\bar{q}_A \tau^a q^A \big) \epsilon_{ij} = 0, \qquad (a = 1,2,3); \\
&\frac{1}{2g_1} F^{(8)}_{ij} \pm \frac{g_1}{4\sqrt{3}} \left(|q^A|^2 - 2\xi \right) \epsilon_{ij} = 0, \\
&\nabla_i \, q^A + i \varepsilon \epsilon_{ij} \nabla_j \, q^A = 0, \qquad A = 1,2.
\end{aligned} \tag{3.7}
$$

follow from the last formula. The equations (3.7) have abelian (n,k) solutions of the type (where n,k are integers) studied in [33]

$$
\begin{aligned}
q^{kA} &= \begin{pmatrix} e^{i\,n\,\varphi} \phi_1(r) & 0 \\ 0 & e^{i\,k\,\varphi} \phi_2(r) \end{pmatrix}, \\
A^3_i(x) &= -\varepsilon \epsilon_{ij} \, \frac{x_j}{r^2} \; \left((n-k) - f_3(r) \right), \\
A^8_i(x) &= -\sqrt{3} \; \varepsilon \epsilon_{ij} \, \frac{x_j}{r^2} \; \left((n+k) - f_8(r) \right)
\end{aligned} \tag{3.8}
$$

where $\phi_1(r)$, $\phi_2(r)$, $f_3(r)$, $f_8(r)$ are profile functions with appropriate boundary consitions.

The crucial observation is that the system (3.6) has an exact $SU(2)_{C+F}$ symmetry, which is neither broken by the interactions nor by the squark VEVS. However, an individual vortex configuration breaks it as $SU(2)_{C+F} \to U(1)$ therefore the vortex acquires zero modes parametrizing

$$
\frac{SU(2)}{U(1)} \sim CP^1 \sim S^2.
$$

For instance, minimum vortices of generic orientation (all degenerate) can be explicitly constructed as

$$
\begin{aligned}
q^{kA} = U \begin{pmatrix} e^{i\varphi} \phi_1(r) & 0 \\ 0 & \phi_2(r) \end{pmatrix} U^{-1} &= e^{\frac{i}{2}\varphi(1+n^a\tau^a)} \, U \begin{pmatrix} \phi_1(r) & 0 \\ 0 & \phi_2(r) \end{pmatrix} U^{-1}, \\
A_i(x) = U[-\frac{\tau^3}{2} \, \epsilon_{ij} \, \frac{x_j}{r^2} \, [1 - f_3(r)]]U^{-1} &= -\frac{1}{2} \, n^a \tau^a \epsilon_{ij} \, \frac{x_j}{r^2} \, [1 - f_3(r)], \\
A^8_i(x) &= -\sqrt{3} \; \epsilon_{ij} \, \frac{x_j}{r^2} \, [1 - f_8(r)],
\end{aligned} \tag{3.9}
$$

where U is an $SU(2)$ matrix, which smoothly interpolate between the abelian $(1,0)$ and $(0,1)$ vortices. Explicitly, if $n^a = (\sin\alpha\cos\beta, \sin\alpha\sin\beta, \cos\alpha)$, the rotation matrix is given by $U = \exp -i\beta\,\tau_3/2 \cdot \exp -i\alpha\,\tau_2/2$.

The (massive) non-abelian monopoles resulting from the gauge symmetry breaking $SU(3) \to SU(2) \times U(1)/Z_2$ by the adjoint Φ VEV, are confined by these non-abelian monopoles.

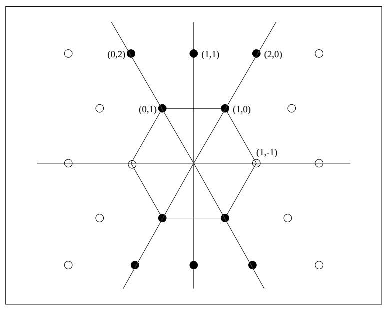


Figure 1: The spectrum of the abelian vortices in the $U(1) \times U(1)$ gauge theory in the Higgs phase [33].

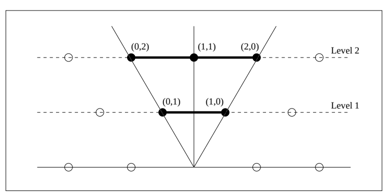


Figure 2: The reduced spectrum of the non-abelian vortices in the $SU(2) \times U(1)/\mathbb{Z}_2$ gauge theory (Eq.(3.6)) in the Higgs phase [32].

3.9 Subtle are (also) non-abelian vortices :

The reduction of the vortex spectrum - meson spectrum (see Figures above) - is due to the topology change

$$\Pi_1\left(\frac{U(1) \times U(1)}{\mathbf{Z}_2}\right) = \mathbf{Z}^2 \quad \to \quad \Pi_1\left(\frac{SU(2) \times U(1)}{\mathbf{Z}_2}\right) = \mathbf{Z};$$

which occurs in the limit of equal masses $m_i \to m$. The transition from the abelian ($m_i \neq m_j$) to the non-abelian ($m_i = m$) superconductivity is here reliably and quantum mechanically analyzed as the $SU(2) \times U(1)$ subgroup is non asymptotically free for

$n_f = 4$ or $n_f = 5$. *Note that the quantum behavior of the non-abelian vortices also depends crucially on the massless fermions present in the underlying theory [32].* For instance, in the $N = 2$, $SU(3)$ theory with n_f less or equal to 3, there are no quantum vacua with unbroken $SU(2)$ gauge group.

Existence of degenerate, non-abelian vortices which continuously interpolate from $(1, 0)$ to $(0, 1)$ vortices, imply a corresponding $SU(2)$ doublet of monopoles transforming continuously between them, as the latter act as the sources (or sink) of the former (see Figure below) when the full $SU(3)$ interactions are taken into account.

The dynamics of vortex zero modes can be shown to be equivalent to the two-dimensional $O(3) = \mathbf{CP}^1$ sigma model ($\mathbf{n} \to \mathbf{n}(z, t)$):

$$S_\sigma^{(1+1)} = \beta \int dz\, dt\, \frac{1}{2}\, (\partial\, n^a)^2 + \text{fermions}.$$

It is dual [34, 35] to a chiral theory with two vacua. The exact $SU(2)_{C+F}$ symmetry is not spontaneously broken. The dual ($N = 1$) $SU(2)$ theory is in confinement phase and has, correctly, two vacua (Witten index).

The whole picture generalizes naturally to the sytem with the symmetry breaking

$$SU(N) \to \frac{SU(N-1) \times U(1)}{\mathbf{Z}_{N-1}} \to \emptyset \tag{3.10}$$

with $2N > N_f \geq 2(N-1)$ flavors. The system at intermediate scales has vortices with $2(N-2)$-parameter family of zero modes representing

$$\frac{SU(N-1)}{SU(N-2) \times U(1)} \sim \mathbf{CP}^{N-2}.$$

The analysis of [32] was made at large m (large $\langle\phi\rangle$) where the system is semiclassical. It is the existence of this large disparity of scales ($m \gg \sqrt{\xi}$) which allows us to treat approximately both the BPS monopoles and vortices as stable configurations. This and other questions on non-abelian monopoles and vortices will be expounded in a forthcoming paper [36].

Though more difficult to analyze, the situation at small m where the non-abelian monopoles condense and the quarks are confined by non-abelian chromoelectric vortices, is related smoothly to the non-abelian superconductor studied here, via holomorphic dependence of the physics on m and though the isomonodromy (in which quarks become monopoles and vice versa).

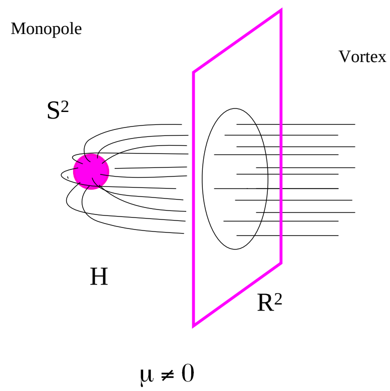


Figure 3: A minimum non-abelian vortex ends on a minimum non-abelian monopole with an exactly matched flux [36].

3.10 Lessons from $N = 2$ SQCD

Summarizing, softly broken $N = 2$, $SU(n_c)$ gauge theories with n_f quarks with $m = 0$, show different types of confining vacua:

1. $r = 0, 1$ vacua are described by weakly coupled abelian monopoles;
2. $r < \frac{n_f}{2}$: The ground state is a non-abelian superconductor; non-abelian monopoles condense and confines the quarks;
3. $r = \frac{n_f}{2}$ is a boundary case: the ground state is a deformed SCFT, with strongly coupled non-abelian monopoles and dyons.

In the $USp(2n_c)$ and $SO(n_c)$ gauge theories all confining vacua are of the third type. Both at generic r - vacua and at the SCFT ($r = \frac{n_f}{2}$) vacua of $SU(n_c)$ SQCD, non-abelian monopoles condense as

$$\langle M_\alpha^i \rangle = \delta_\alpha^i \, v \neq 0, \qquad (\alpha = 1, 2, \ldots, r; \quad i = 1, 2, \ldots, n_f)$$

("Color-Flavor-Locked phase"). There are some indications that a similar result holds in the $r = \frac{n_f}{2}$, almost conformal vacua [27].

4 Hints for QCD

What can one learn from these studies for QCD? First of all, dynamical abelianization neither is observed in the real world nor is believed to occur in QCD. On the other hand,

QCD with n_f flavors and its possible dual have the beta function coefficients ($\tilde{n}_c = 2, 3$, $n_f = 2, 3$) such that

$$b_0 = 11\, n_c - 2\, n_f \quad vs \quad \tilde{b}_0 = 11\, \tilde{n}_c - n_f,$$

where we assumed that in the standard QCD, the flavor carrying monopoles are scalars. Because of the large coefficient (eleven) in front of the color multiplicity, a sign flip (weakly-coupled non-abelian monopoles) is hardly possible, even though in nonsupersymmetric theories higher loop contributions are also important. These two facts, together, leave the option of a strongly-interacting non-abelian superconductor, as in the almost superconformal vacua in the $N = 2$ gauge theories, as the most likely picture of the ground state of QCD.

Taking a more detailed hint from supersymmetric models [24] one might assume that non-abelian magnetic monopoles of QCD condense in a color-flavor-diagonal form

$$\langle M^i_{L,\alpha} \rangle = \delta^i_\alpha\, v_R \neq 0, \quad \langle M^\alpha_{R,i} \rangle = \delta^\alpha_i\, v_L \neq 0,$$

($\alpha = 1, 2, \ldots \tilde{n}_c; i = 1, 2, \ldots n_f$). As they are strongly coupled, a better physical picture might be

$$\langle M^i_{L,\alpha}\, M^\alpha_{R,j} \rangle = \text{const.}\, \delta^i_j \neq 0;$$

which yields for $\tilde{n}_c = 2$, $n_f = 2$ the correct symmetry breaking pattern

$$G_F = SU_L(2) \times SU_R(2) \Rightarrow SU_V(2),$$

observed in Nature. $U_V(1)$ (baryon number) is not broken spontaneously in the real world, therefore the massless non-abelian monopoles would have to possess a vanishing quark number. Intriguingly, precisely such a phenomenon occurs in supersymmetric theories as a result of nonperturbative renormalization [21].

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